

Kinetic analysis of spin current contribution to spectrum of electromagnetic waves in spin-1/2 plasma, Part I: Dielectric permeability tensor for magnetized plasmas

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The dielectric permeability tensor for spin polarized plasmas is derived in terms of the spin-1/2 quantum kinetic model in six-dimensional phase space. Expressions for the distribution function and spin distribution function are derived in linear approximations on the path of dielectric permeability tensor derivation. The dielectric permeability tensor is derived the spin-polarized degenerate electron gas. It is also discussed at the finite temperature regime, where the equilibrium distribution function is presented by the spin-polarized Fermi-Dirac distribution. Consideration of the spin-polarized equilibrium states opens possibilities for the kinetic modeling of the thermal spin current contribution in the plasma dynamics.

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I. INTRODUCTION

Spin effects in plasmas were introduced via hydrodynamic formalism [1–5]. Later, a kinetic model was reinstalled in [6] recapturing results of [7–9], where the phase space was extended up to eight dimensions to include the spin dependence of the distribution function of particles with fixed module of the spin. Both approaches attract attention of a lot of researchers. Another kinetic model of spin-1/2 particles was reinstalled in [10–15] recapturing results of [16–21], where the distribution functions were considered in the traditional six dimensional phase space. Experimental conformation of results obtained by Dyson [17] is found in [22]. In this model, the spin evolution appears via presence of the vector distribution function—the spin distribution function in addition to the traditional distribution function. A quantum-relativistic kinetic equations were derived applying the Wigner distribution function [23]. It was derived from a single-particle Dirac equation. Hence, it may neglect relativistic interparticle effects arising between particles of comparable masses [24].

Next step in the development of models for the spin-1/2 quantum plasmas was the derivation of the separated spin evolution quantum hydrodynamics [25], [26] (see [27] for a generalized model with exchange interaction) and separated spin evolution quantum kinetics [28]. In this model the electrons are separated on two subspecies: electrons with spin-up and electrons with spin-down [25, 29]. The separated spin evolution quantum hydrodynamics is derived from the single particle Pauli equation [25]. A many-particle derivation was suggested for the separated spin evolution quantum kinetic derivation [28]. The derivation based on the Pauli equation discover a specific structure of the spin-spin interaction

force field. Moreover, it demonstrates unconservation of the particle number in each subspecies. Different behavior of spin-up and spin-down electrons required different pressure. Corresponding equation of state was presented in [25]. All these features of the separated spin evolution are obtained in Refs. [25], [26] and [28], while a two fluid model of electrons was mentioned in literature earlier [29], [30], [31]. Incompleteness of earlier model bound to incorrect coefficients in spin depending terms [32].

Spin evolution leads to the thermal part of the spin flux or the spin current. The thermal part of the spin current is an analog of the pressure existing in the Euler equation. It is expected that this spin current considerably affects spin properties of plasmas. However, its analysis requires an equation of state. Corresponding equation of state is found recently with application of the separated spin evolution quantum hydrodynamics [33]:

$$n(\partial_t + \mathbf{u} \cdot \nabla)\boldsymbol{\mu} - \frac{\hbar}{2m\mu_e}\partial^\beta[n\boldsymbol{\mu} \times \partial^\beta\boldsymbol{\mu}] + \mathfrak{S} = \frac{2\mu_e}{\hbar}n[\boldsymbol{\mu} \times \mathbf{B}], \quad (1)$$

where $\boldsymbol{\mu} = \mathbf{M}/n$, $\mathbf{M} = \mathbf{M}(\mathbf{r}, t)$ is the magnetization of electron gas, $n = n(\mathbf{r}, t)$ is the concentration of particles, $\mathbf{u}(\mathbf{r}, t)$ is the velocity field, \mathbf{B} is the magnetic field, \hbar is the reduced Planck constant, m is the mass of particle, μ_e is the magnetic moment of particle, ∇ and ∂^β are the vector and tensor notations for the spatial derivatives, and \mathfrak{S} is the divergence of the thermal part of the spin current, its explicit form is found for the degenerate electron gas [33]:

$$\mathfrak{S}_P = \frac{(3\pi^2)^{2/3}\hbar}{m}(n_\uparrow^{2/3} - n_\downarrow^{2/3})[\mathbf{M}, \mathbf{e}_z], \quad (2)$$

with $n_\uparrow = n - M_z / |\mu_e|$, $n_\downarrow = n + M_z / |\mu_e|$, where μ_e is the magnetic moment of electron, and subindex P shows that equation (2) is derived from the non-linear Pauli equation, indexes \uparrow and \downarrow refer to the spin-up and

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spin-down states, correspondingly. It is an analog of the Fermi pressure. Therefore, it is called the Fermi spin current. It leads to the modification of spin-plasma wave spectrum [33]. The cut-off frequency of the spin waves is modified by the Fermi spin current. If cut-off frequency of the spin-plasma wave is larger than the plasma frequency the linear interaction of the spin-plasma wave and the ordinary wave appears. Polarization of spin-plasma wave propagating parallel to the external magnetic field changes as well. Hence, a left-hand polarized wave appears instead of the right-hand polarized [33]. Modifications of spectrum and cut-off frequency appear at the perpendicular propagation either. Similar equation of state was derived from the separated spin evolution quantum kinetics as a moment of the spin distribution function [28]: $J_K^{xx} = J_K^{yy} = \cdot 3\pi\hbar\mu_e(6\pi^2)^{1/3}(n_{\uparrow}^{4/3} - n_{\downarrow}^{4/3})/32m$ and $J_K^{xy} = J_K^{xz} = J_K^{yx} = J_K^{yz} = J_K^{zx} = J_K^{zy} = J_K^{zz} = 0$, where subindex K demonstrates that this result is obtained from kinetic model being defined as follows $J_K^{\alpha\beta} = \mu_e \int S_0^\alpha(\mathbf{p})v^\beta dp$. Separate spin evolution affects extraordinary waves directly via the difference of the Fermi pressures for spin-up and spin-down electrons [34]. While, the spin evolution and the Fermi spin current do not affect these waves.

Existence of the Fermi spin current is closely related to difference of the Fermi pressure for the spin-up and spin-down electrons [33]. Difference of the Fermi pressures itself reveals in a modification (an increase) of the pressure contribution in the properties of longitudinal waves. Difference of the Fermi pressures presented in the two fluid model of electron gas leads to extra phenomena: a pair of bulk spin-electron acoustic waves [25], [26], spin-electron acoustic soliton [27], surface spin-electron acoustic wave [35], a pair of bulk spin-electron-positron acoustic waves in addition to the pair of the bulk spin-electron acoustic waves existing in electron-positron-ion plasmas [36], spin-electron acoustic soliton and spin-electron-positron acoustic soliton in electron-positron-ion plasmas [37]. All these waves are longitudinal waves.

The Fermi spin current contributes to the transverse waves [33]. Results of application of the Fermi spin current require the proper generalization by means of a kinetic model. This paper is devoted to the development of corresponding kinetic model. Presenting derivation of the dielectric permeability tensor for spin polarized plasmas is the first step in this direction.

This paper is organized as follows. In Sec. II, basic nonrelativistic kinetic equations for spin-1/2 plasmas are presented. In Sec. III, linearized kinetic equations and their solutions for the isotropic equilibrium distribution function are obtained. In Sec. IV, the dielectric permeability tensor is found. It is presented in general form and for the spin-polarized Fermi step distribution function. In Sec. V, a summary of the obtained results is presented.

II. QUANTUM KINETIC MODEL FOR SPIN-1/2 PLASMAS

Different quantum kinetic approaches have been developed [6, 13–15], [28], [38–44]. Some of them do not include the spin evolution [38, 39, 41–44], but consider the exchange part of the Coulomb interaction [39], [44], or consider non-ideal plasmas with strong interaction [40], [41].

The kinetic equation for the distribution function $f(\mathbf{r}, \mathbf{p}, t)$ in spin-1/2 plasmas appears as follows [13], [15]:

$$\begin{aligned} \partial_t f + \mathbf{v} \cdot \nabla_{\mathbf{r}} f + q_e \left(\mathbf{E}_{ext} + \frac{1}{c} \mathbf{v} \times \mathbf{B}_{ext} \right) \cdot \nabla_{\mathbf{p}} f + \mu_e (\nabla_{\mathbf{r}}^\beta B_{ext}^\alpha) \nabla_{\mathbf{p}}^\beta S^\alpha \\ - q_e^2 \int \nabla_{\mathbf{r}} G(\mathbf{r}, \mathbf{r}') \cdot \nabla_{\mathbf{p}} f_2(\mathbf{r}, \mathbf{p}, \mathbf{r}', \mathbf{p}', t) d\mathbf{r}' d\mathbf{p}' \\ - \mu_e^2 \int (\nabla_{\mathbf{r}}^\alpha G^{\beta\gamma}(\mathbf{r}, \mathbf{r}')) \nabla_{\mathbf{p}}^\alpha S_2^{\beta\gamma}(\mathbf{r}, \mathbf{p}, \mathbf{r}', \mathbf{p}', t) d\mathbf{r}' d\mathbf{p}' = 0, \end{aligned} \quad (3)$$

where $G(\mathbf{r}, \mathbf{r}') = 1/|\mathbf{r} - \mathbf{r}'|$ is the Green function of Coulomb interaction, $G^{\alpha\beta}(\mathbf{r}, \mathbf{r}') = \partial^\alpha \partial^\beta (1/|\mathbf{r} - \mathbf{r}'|) + 4\pi \delta^{\alpha\beta} \delta(\mathbf{r} - \mathbf{r}')$ is the Green function of spin-spin interaction, $\nabla_{\mathbf{p}}$ is the derivative on the momentum $\mathbf{p} = m\mathbf{v}$. It contains the two-particle distribution function $f_2(\mathbf{r}, \mathbf{p}, \mathbf{r}', \mathbf{p}', t)$ in the term describing the Coulomb interaction. The spin distribution function $\mathbf{S}(\mathbf{r}, \mathbf{p}, t)$ arises in the term describing interaction of the spins (the magnetic moments) with the external magnetic field \mathbf{B}_{ext} . The two-particle spin distribution function $S^{\alpha\beta}(\mathbf{r}, \mathbf{p}, \mathbf{r}', \mathbf{p}', t)$, which is a second rank tensor, appears in the term describing the spin-spin interaction. The two-particle distribution functions can be reduced to the one-particle distribution functions in the self-consistent (mean-field) approximation. However, equation (3) requires an additional equation. The equation of evolution of the spin-distribution function $\mathbf{S}(\mathbf{r}, \mathbf{p}, t)$, which arrears as follows [13], [15]:

$$\begin{aligned} \partial_t S^\alpha + \mathbf{v} \cdot \nabla_{\mathbf{r}} S^\alpha \\ + q_e \left(\mathbf{E}_{ext} + \frac{1}{c} \mathbf{v} \times \mathbf{B}_{ext} \right) \cdot \nabla_{\mathbf{p}} S^\alpha + \mu_e (\nabla_{\mathbf{r}}^\beta B_{ext}^\alpha) \nabla_{\mathbf{p}}^\beta f \\ + q_e^2 \int \nabla_{\mathbf{r}} G(\mathbf{r}, \mathbf{r}') \cdot \nabla_{\mathbf{p}} M_2^\alpha(\mathbf{r}, \mathbf{p}, \mathbf{r}', \mathbf{p}', t) d\mathbf{r}' d\mathbf{p}' \\ - \mu_e^2 \int (\nabla_{\mathbf{r}}^\gamma G^{\alpha\beta}(\mathbf{r}, \mathbf{r}')) \nabla_{\mathbf{p}}^\gamma N_2^\beta(\mathbf{r}, \mathbf{p}, \mathbf{r}', \mathbf{p}', t) d\mathbf{r}' d\mathbf{p}' \\ - \frac{2\mu_e}{\hbar} \varepsilon^{\alpha\beta\gamma} \left(S^\beta(\mathbf{r}, \mathbf{p}, t) B_{ext}^\gamma(\mathbf{r}, t) \right) \end{aligned}$$

$$+ \mu_e \int G^{\gamma\delta}(\mathbf{r}, \mathbf{r}') S_2^{\beta\delta}(\mathbf{r}, \mathbf{p}, \mathbf{r}', \mathbf{p}', t) d\mathbf{r}' d\mathbf{p}') = 0. \quad (4)$$

Equation (4) contains the mixed spin-number of particles two-particle distribution function $M_2^\alpha(\mathbf{r}, \mathbf{p}, \mathbf{r}', \mathbf{p}', t)$ and the number of particles-spin two-particle distribution function $N_2^\beta(\mathbf{r}, \mathbf{p}, \mathbf{r}', \mathbf{p}', t)$. The quantum terms containing the Planck constant and higher derivatives of the distribution functions are neglected in equations (3) and (4).

The two-particle distribution functions containing in the kinetic equations have the following representation in the self-consistent field approximation

$$f_2(\mathbf{r}, \mathbf{p}, \mathbf{r}', \mathbf{p}', t) = f(\mathbf{r}, \mathbf{p}, t) f(\mathbf{r}', \mathbf{p}', t), \quad (5)$$

$$S_2^{\alpha\beta}(\mathbf{r}, \mathbf{p}, \mathbf{r}', \mathbf{p}', t) = S^\alpha(\mathbf{r}, \mathbf{p}, t) S^\beta(\mathbf{r}', \mathbf{p}', t), \quad (6)$$

$$M_2^\alpha(\mathbf{r}, \mathbf{p}, \mathbf{r}', \mathbf{p}', t) = S^\alpha(\mathbf{r}, \mathbf{p}, t) f(\mathbf{r}', \mathbf{p}', t), \quad (7)$$

and

$$N_2^\beta(\mathbf{r}, \mathbf{p}, \mathbf{r}', \mathbf{p}', t) = f(\mathbf{r}, \mathbf{p}, t) S^\beta(\mathbf{r}', \mathbf{p}', t). \quad (8)$$

Equations (3) and (4) are obtained at the neglecting the quantum terms explicitly containing the Planck constant in the kinetic equations (for details see [13], [15]).

It is assumed in equation (3) that the kinetic current is reduced to the distribution function $\mathbf{J}(\mathbf{r}, \mathbf{p}, t) = \mathbf{p} f(\mathbf{r}, \mathbf{p}, t)$ [13] (see equations 12 and 13). Similar approximation is made for the kinetic spin current $J^{\alpha\beta}(\mathbf{r}, \mathbf{p}, t) = p^\alpha S^\beta(\mathbf{r}, \mathbf{p}, t)$ in the second term in the kinetic equation (4) [13] (see equations 36 and 37).

As the result we have next set of equations describing the evolution of spin-1/2 plasma in the self-consistent field approximation [13], [14], [15]

$$\partial_t f + \mathbf{v} \cdot \nabla_{\mathbf{r}} f + q_e \left(\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right) \cdot \nabla_{\mathbf{p}} f + \nabla_{\mathbf{r}}^\alpha B^\beta \cdot \nabla_{\mathbf{p}}^\alpha S^\beta = 0, \quad (9)$$

and

$$\begin{aligned} \partial_t S^\alpha + \mathbf{v} \cdot \nabla_{\mathbf{r}} S^\alpha + q_e \left(\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right) \cdot \nabla_{\mathbf{p}} S^\alpha \\ + \nabla_{\mathbf{r}}^\beta B^\alpha \cdot \nabla_{\mathbf{p}}^\beta f - \frac{2\mu_e}{\hbar} \varepsilon^{\alpha\beta\gamma} S^\beta B^\gamma = 0. \end{aligned} \quad (10)$$

Internal electromagnetic field in equations (9) and (10) appears in the electro-magneto-static limit, the electric field

$$\mathbf{E}_{int} = q_e \int \nabla G(\mathbf{r}, \mathbf{r}') f(\mathbf{r}', \mathbf{p}, t) d\mathbf{r}' d\mathbf{p} \quad (11)$$

satisfying quasi-static equations $\nabla \times \mathbf{E} = 0$ and

$$\nabla \cdot \mathbf{E} = 4\pi q_e \int f(\mathbf{r}, \mathbf{p}, t) d\mathbf{p}, \quad (12)$$

and the magnetic field

$$B_{int}^\alpha = \mu_e \int G^{\alpha\beta}(\mathbf{r}, \mathbf{r}') S^\beta(\mathbf{r}', \mathbf{p}, t) d\mathbf{r}' d\mathbf{p} \quad (13)$$

satisfying the following quasi-static equations $\nabla \cdot \mathbf{B} = 0$ and

$$\nabla \times \mathbf{B} = 4\pi \mu_e \nabla \times \int \mathbf{S}(\mathbf{r}, \mathbf{p}, t) d\mathbf{p}. \quad (14)$$

To study the plasma dynamics we need to generalize equations (11)-(14) arising at the non-relativistic derivation up to the full set of the Maxwell equations

$$\nabla \cdot \mathbf{E} = 4\pi \rho, \quad \nabla \times \mathbf{E} = -\frac{1}{c} \partial_t \mathbf{B}, \quad \nabla \cdot \mathbf{B} = 0, \quad (15)$$

and

$$\nabla \times \mathbf{B} = \frac{1}{c} \partial_t \mathbf{E} + \frac{4\pi}{c} \mathbf{j} + 4\pi \nabla \times \mathbf{M}, \quad (16)$$

where $\rho = q_e \int f(\mathbf{r}, \mathbf{p}, t) d\mathbf{p} + q_i n_{oi}$, $\mathbf{j} = q_e \int \mathbf{v} f(\mathbf{r}, \mathbf{p}, t) d\mathbf{p}$, $\mathbf{M} = \mu_e \int \mathbf{S}(\mathbf{r}, \mathbf{p}, t) d\mathbf{p}$ is the magnetization.

Existing kinetic research shows that spin effects leads to damping effects for the electrostatic wave modes [45], existing of spin waves [6], and splitting of Bernstein modes (each mode splits on three branches at the account of the anomalous part of magnetic moment of electrons) [46]. Kinetic gives an advanced description in compare with hydrodynamics. However, some effects, which do not enter result of traditional hydrodynamics, can be found in extended hydrodynamic models [47], [48].

III. LINEARIZED KINETIC EQUATIONS AND SOLUTIONS FOR DISTRIBUTION FUNCTIONS

For the derivation of the dielectric permeability tensor we need to consider linear on small perturbations kinetic equations. Assuming that in equilibrium we have non-zero $f_0(p)$, $\mathbf{B}_0 = B_0 \mathbf{e}_z$, $\mathbf{S}_0 = S_0(p) \mathbf{e}_z$, with $p = |\mathbf{p}|$, we find from kinetic equations (3) and (4) the following linearized Fourier transformed kinetic equations

$$\begin{aligned} -i\omega \delta f + i\mathbf{v} \cdot \mathbf{k} \delta f + \frac{q_e}{c} B_0 (\mathbf{v} \times \mathbf{e}_z) \cdot \nabla_{\mathbf{p}} \delta f \\ + q_e \delta \mathbf{E} \cdot \nabla_{\mathbf{p}} f_0 + i\mu_e \delta B_z \mathbf{k} \cdot \nabla_{\mathbf{p}} S_0 = 0, \end{aligned} \quad (17)$$

and

$$\begin{aligned} -i\omega \delta \mathbf{S} + i(\mathbf{v} \cdot \mathbf{k}) \delta \mathbf{S} + \frac{q_e}{c} B_0 ((\mathbf{v} \times \mathbf{e}_z) \cdot \nabla_{\mathbf{p}}) \delta \mathbf{S} + i\mu_e (\mathbf{k} \cdot \nabla_{\mathbf{p}}) f_0 \delta \mathbf{B} \\ + q_e (\delta \mathbf{E} \cdot \nabla_{\mathbf{p}}) \mathbf{S}_0 + \frac{2\mu_e}{\hbar} (\mathbf{B}_0 \times \delta \mathbf{S} - \mathbf{S}_0 \times \delta \mathbf{B}) = 0, \end{aligned} \quad (18)$$

where the wave vector has the following structure $\mathbf{k} = \{k_x, 0, k_z\}$ that corresponds to the oblique propagation

of waves relatively to the external magnetic field. It corresponds to the isotropic equilibrium distribution functions. More general case of equilibrium distribution functions and solutions for linear distribution functions are presented in Appendix A.

As it follows from equation (18) projections of the spin distribution function \mathbf{S} satisfy the following linear equations

$$\begin{aligned} & \Omega_e \partial_\varphi \delta S_x + \imath(\omega - \mathbf{k} \cdot \mathbf{v}) \delta S_x + \Omega_\mu \delta S_y \\ &= \imath \mu_e (\mathbf{k} \cdot \nabla_{\mathbf{p}}) f_0 \delta B_x + \frac{2\mu_e}{\hbar} S_{0z} \delta B_y, \end{aligned} \quad (19)$$

and

$$\begin{aligned} & -\Omega_\mu \delta S_x + \Omega_e \partial_\varphi \delta S_y + \imath(\omega - \mathbf{k} \cdot \mathbf{v}) \delta S_y \\ &= \imath \mu_e (\mathbf{k} \cdot \nabla_{\mathbf{p}}) f_0 \delta B_y - \frac{2\mu_e}{\hbar} S_{0z} \delta B_x, \end{aligned} \quad (20)$$

where δS_x and δS_y are bound to each other; and

$$\begin{aligned} & \Omega_e \partial_\varphi \delta S_z + \imath(\omega - \mathbf{k} \cdot \mathbf{v}) \delta S_z \\ &= q_e (\delta \mathbf{E} \cdot \nabla_{\mathbf{p}}) S_{0z} + \imath \mu_e (\mathbf{k} \cdot \nabla_{\mathbf{p}}) f_0 \delta B_z \end{aligned} \quad (21)$$

which is independent from perturbations of other distribution functions. The following notations are used for the charge cyclotron frequency $\Omega_e = q_e B_0 / mc$ and the magnetic moment cyclotron frequency $\Omega_\mu = 2\mu_e B_0 / \hbar$. They are equal to each other if the anomalous part of magnetic moment of electron is neglected. At the transition from equations (17) and (18) to equations (61)-(63) we used that $(\mathbf{v} \times \mathbf{e}_z) \cdot \partial_{\mathbf{p}} = (1/m) \partial_\varphi$.

Equation for δf (17) is independent from other equations. Equation for δS_z (63) is independent either. Equations for δS_x (61) and δS_y (62) make a set of equations and should be solved together.

Comparing equation (17) with spinless case it can be seen that it differs by a single term on the right-hand side. Therefore, solution of equation (17) can be found in the traditional form

$$\begin{aligned} \delta f &= \frac{1}{\Omega_e} \int_{C_0}^\varphi \left(q_e (\mathbf{v} \cdot \delta \mathbf{E}) |_{\varphi'} \frac{\partial f_0}{\partial \varepsilon} + \imath \mu_e (\mathbf{k} \cdot \mathbf{v}) |_{\varphi'} \delta B_z \frac{\partial S_{0z}}{\partial \varepsilon} \right) \times \\ & \times \exp \left(\imath \int_{\varphi}^{\varphi'} \frac{1}{\Omega_e} (\omega - \mathbf{k} \cdot \mathbf{v} |_{\varphi''}) d\varphi'' \right) d\varphi', \end{aligned} \quad (22)$$

but containing two terms under integral instead of one. In formula (22) and below symbol $|_{\varphi'}$ means that it is a function of φ' . Above, the differentiation on the momentum is replaced by the differentiation on energy $\varepsilon = p^2/2m$, $\nabla_{\mathbf{p}} = \mathbf{v} \partial_\varepsilon$.

Equation (63) can be solved similarly. As the result, the following solution is found

$$\delta S_z = \frac{1}{\Omega_e} \int_{C_3}^\varphi \left(q_e (\mathbf{v} \cdot \delta \mathbf{E}) |_{\varphi'} \frac{\partial S_{0z}}{\partial \varepsilon} + \imath \mu_e (\mathbf{k} \cdot \mathbf{v}) |_{\varphi'} \delta B_z \frac{\partial f_0}{\partial \varepsilon} \right) \times$$

$$\times \exp \left(\imath \int_{\varphi}^{\varphi'} \frac{1}{\Omega_e} (\omega - \mathbf{k} \cdot \mathbf{v} |_{\varphi''}) d\varphi'' \right) d\varphi'. \quad (23)$$

Spin part of the distribution functions appears via the magnetic field perturbations, which can be represented via the electric field for the further derivation of the dielectric permeability tensor $\delta B_x = -k_z c \delta E_y / \omega$, $\delta B_y = c(k_z \delta E_x - k_x \delta E_z) / \omega$, $\delta B_z = k_x c \delta E_y / \omega$.

Next, the following anzac can be used for simplification of set of equations (61) and (62)

$$\delta S_x = P(\varphi) \exp \left(-\imath \int_C^\varphi \frac{1}{\Omega_e} (\omega - \mathbf{k} \cdot \mathbf{v} |_{\varphi'}) d\varphi' \right), \quad (24)$$

and

$$\delta S_y = R(\varphi) \exp \left(-\imath \int_C^\varphi \frac{1}{\Omega_e} (\omega - \mathbf{k} \cdot \mathbf{v} |_{\varphi'}) d\varphi' \right). \quad (25)$$

It gives the following equations for functions $P(\varphi)$ and $R(\varphi)$:

$$\begin{aligned} \Omega_e \partial_\varphi P(\varphi) + \Omega_\mu R(\varphi) &= \exp \left(\imath \int_C^\varphi \frac{\omega - \mathbf{k} \cdot \mathbf{v} |_{\varphi'}}{\Omega_e} d\varphi' \right) \times \\ & \times \left(\imath \mu_e (\mathbf{k} \cdot \nabla_{\mathbf{p}}) f_0 \delta B_x + \frac{2\mu_e}{\hbar} S_{0z} \delta B_y \right), \end{aligned} \quad (26)$$

and

$$\begin{aligned} -\Omega_\mu P(\varphi) + \Omega_e \partial_\varphi R(\varphi) &= \exp \left(\imath \int_C^\varphi \frac{\omega - \mathbf{k} \cdot \mathbf{v} |_{\varphi'}}{\Omega_e} d\varphi' \right) \times \\ & \times \left(\imath \mu_e (\mathbf{k} \cdot \nabla_{\mathbf{p}}) f_0 \delta B_y - \frac{2\mu_e}{\hbar} S_{0z} \delta B_x \right). \end{aligned} \quad (27)$$

The structure of functions $P(\varphi)$ and $R(\varphi)$ can be found by solving homogeneous part of equations (26) and (27):

$$P(\varphi) = \imath p(\varphi) \exp \left(\frac{\imath \Omega_\mu}{\Omega_e} \varphi \right) - \imath r(\varphi) \exp \left(-\frac{\imath \Omega_\mu}{\Omega_e} \varphi \right), \quad (28)$$

and

$$R(\varphi) = p(\varphi) \exp \left(\frac{\imath \Omega_\mu}{\Omega_e} \varphi \right) + r(\varphi) \exp \left(-\frac{\imath \Omega_\mu}{\Omega_e} \varphi \right). \quad (29)$$

Equations for $p(\varphi)$ and $r(\varphi)$ can be obtained at the substituting of expressions (28) and (29) into equations (26) and (27):

$$\imath \partial_\varphi p(\varphi) \cdot \exp \left(\frac{\imath \Omega_\mu}{\Omega_e} \varphi \right) - \imath \partial_\varphi r(\varphi) \cdot \exp \left(-\frac{\imath \Omega_\mu}{\Omega_e} \varphi \right) = \Pi_x, \quad (30)$$

where

$$\Pi_x = \frac{1}{\Omega_e} \exp \left(\imath \int_C^\varphi \frac{1}{\Omega_e} (\omega - \mathbf{k} \cdot \mathbf{v} |_{\varphi'}) d\varphi' \right) \times$$

$$\times \left(\imath \mu_e (\mathbf{k} \cdot \nabla_{\mathbf{p}}) f_0 \delta B_x + \frac{2\mu_e}{\hbar} S_{0z} \delta B_y \right), \quad (31) \quad \text{and}$$

and

$$\partial_{\varphi} p(\varphi) \cdot \exp\left(\frac{\imath \Omega_{\mu}}{\Omega_e} \varphi\right) + \partial_{\varphi} r(\varphi) \cdot \exp\left(-\frac{\imath \Omega_{\mu}}{\Omega_e} \varphi\right) = \Pi_y, \quad (32)$$

where

$$\begin{aligned} \Pi_y &= \frac{1}{\Omega_e} \exp\left(\imath \int_C^{\varphi} \frac{1}{\Omega_e} (\omega - \mathbf{k} \cdot \mathbf{v} |_{\varphi'}) d\varphi'\right) \times \\ &\times \left(\imath \mu_e (\mathbf{k} \cdot \nabla_{\mathbf{p}}) f_0 \delta B_y - \frac{2\mu_e}{\hbar} S_{0z} \delta B_x \right). \end{aligned} \quad (33)$$

Independent equations for functions $p(\varphi)$ and $r(\varphi)$ can be found as combinations of equations (30) and (32):

$$\partial_{\varphi} p(\varphi) = \frac{1}{2\imath} \exp\left(-\frac{\imath \Omega_{\mu}}{\Omega_e} \varphi\right) (\Pi_x + \imath \Pi_y), \quad (34)$$

$$\partial_{\varphi} r(\varphi) = \frac{1}{2\imath} \exp\left(\frac{\imath \Omega_{\mu}}{\Omega_e} \varphi\right) (-\Pi_x + \imath \Pi_y). \quad (35)$$

These equations can be easily integrated. The integration gives the following solutions for functions $p(\varphi)$ and $r(\varphi)$:

$$p(\varphi) = \frac{1}{2\imath} \int_{C_1}^{\varphi} \exp\left(-\frac{\imath \Omega_{\mu}}{\Omega_e} \varphi'\right) (\Pi_x(\varphi') + \imath \Pi_y(\varphi')), \quad (36)$$

and

$$r(\varphi) = \frac{1}{2\imath} \int_{C_2}^{\varphi} \exp\left(\frac{\imath \Omega_{\mu}}{\Omega_e} \varphi'\right) (-\Pi_x(\varphi') + \imath \Pi_y(\varphi')). \quad (37)$$

This calculation leads to the following expressions for the spin distribution functions:

$$\begin{aligned} \delta S_x &= \frac{\mu_e}{2\Omega_e} \left[\int_{C_1}^{\varphi} \exp\left(\frac{\imath \Omega_{\mu}}{\Omega_e} (\varphi - \varphi')\right) \exp\left(\imath \int_{\varphi}^{\varphi'} \frac{\omega - \mathbf{k} \cdot \mathbf{v} |_{\varphi''}}{\Omega_e} d\varphi''\right) \left((\delta B_x + \imath \delta B_y) (\imath \mathbf{k} \cdot \mathbf{v} |_{\varphi'}) \frac{\partial f_0}{\partial \varepsilon} + \frac{2S_{0z}}{\hbar} (\delta B_y - \imath \delta B_x) \right) d\varphi' \right. \\ &\quad \left. + \int_{C_2}^{\varphi} \exp\left(\frac{\imath \Omega_{\mu}}{\Omega_e} (\varphi' - \varphi)\right) \exp\left(\imath \int_{\varphi}^{\varphi'} \frac{\omega - \mathbf{k} \cdot \mathbf{v} |_{\varphi''}}{\Omega_e} d\varphi''\right) \left((\delta B_x - \imath \delta B_y) (\imath \mathbf{k} \cdot \mathbf{v} |_{\varphi'}) \frac{\partial f_0}{\partial \varepsilon} + \frac{2S_{0z}}{\hbar} (\delta B_y + \imath \delta B_x) \right) d\varphi' \right], \end{aligned} \quad (38)$$

and

$$\begin{aligned} \delta S_y &= \frac{\mu_e}{2\imath \Omega_e} \left[\int_{C_1}^{\varphi} \exp\left(\frac{\imath \Omega_{\mu}}{\Omega_e} (\varphi - \varphi')\right) \exp\left(\imath \int_{\varphi}^{\varphi'} \frac{\omega - \mathbf{k} \cdot \mathbf{v} |_{\varphi''}}{\Omega_e} d\varphi''\right) \left((\delta B_x + \imath \delta B_y) (\imath \mathbf{k} \cdot \mathbf{v} |_{\varphi'}) \frac{\partial f_0}{\partial \varepsilon} + \frac{2S_{0z}}{\hbar} (\delta B_y - \imath \delta B_x) \right) d\varphi' \right. \\ &\quad \left. + \int_{C_2}^{\varphi} \exp\left(\frac{\imath \Omega_{\mu}}{\Omega_e} (\varphi' - \varphi)\right) \exp\left(\imath \int_{\varphi}^{\varphi'} \frac{\omega - \mathbf{k} \cdot \mathbf{v} |_{\varphi''}}{\Omega_e} d\varphi''\right) \left((\imath \delta B_y - \delta B_x) (\imath \mathbf{k} \cdot \mathbf{v} |_{\varphi'}) \frac{\partial f_0}{\partial \varepsilon} - \frac{2S_{0z}}{\hbar} (\delta B_y + \imath \delta B_x) \right) d\varphi' \right]. \end{aligned} \quad (39)$$

Solutions (38) and (39) together with solutions (22) and (64) can be used for derivation of dielectric permeability tensor. Constants C_0 , C_1 , C_2 and C_3 are chosen that distribution functions δf and $\delta \mathbf{S}$ are periodic functions of angle φ : $\delta f(\varphi + 2\pi) = \delta f(\varphi)$ and $\delta \mathbf{S}(\varphi + 2\pi) = \delta \mathbf{S}(\varphi)$, so $C_i = \infty$, where $i = 0, 1, 2, 3$.

IV. DIELECTRIC PERMEABILITY TENSOR FOR MAGNETIZED SPIN-1/2 PLASMAS

After Fourier transformation of the Maxwell equations, the magnetic field taken from equation $\mathbf{k} \times \delta \mathbf{E} = \omega \delta \mathbf{B} / c$

is substituted into equation (16). It leads to

$$\left[k^2 \delta^{\alpha\beta} - k^{\alpha} k^{\beta} - \frac{\omega^2}{c^2} \varepsilon^{\alpha\beta}(\omega) \right] \delta E_{\beta} = 0, \quad (40)$$

where the dielectric permeability tensor appears as

$$\begin{aligned} \varepsilon^{\alpha\beta}(\omega) \delta E_{\beta} &= \delta^{\alpha\beta} \delta E_{\beta} + \frac{4\pi \imath q_e}{\omega m} \int p^{\alpha} \delta f d\mathbf{p} \\ &\quad - \frac{4\pi \mu_e c}{\omega} \int \varepsilon^{\alpha\beta\gamma} k^{\beta} \delta S^{\gamma} d\mathbf{p}, \end{aligned} \quad (41)$$

and $\delta^{\alpha\beta}$ is the Kronecker symbol. For further analysis it is useful to distinguish a part of conductivity tensor

caused by the current

$$\sigma_1^{\alpha\beta}(\omega)\delta E_\beta = \frac{q_e}{m} \int p^\alpha \delta f d\mathbf{p} \quad (42)$$

and another part caused by the curl of magnetization

$$\sigma_2^{\alpha\beta}(\omega)\delta E_\beta = i\mu_e c \int \varepsilon^{\alpha\beta\gamma} k^\beta \delta S^\gamma d\mathbf{p}. \quad (43)$$

The standard resolution between $\varepsilon^{\alpha\beta}$ and $\sigma^{\alpha\beta}$ is used: $\varepsilon^{\alpha\beta} = \delta^{\alpha\beta} + (4\pi i/\omega)\sigma^{\alpha\beta}$.

A. General form of dielectric permeability tensor for isotropic distribution functions

An explicit form of the conductivity tensor (42), (43) is presented in Appendix B. After the integration on φ and φ' in equations (67)-(73) (which are presented in Appendix B) we find the following dielectric permeability tensor

$$\begin{aligned} \varepsilon^{\alpha\beta} = & \delta^{\alpha\beta} + \int d\mathbf{p} \sum_{n=-\infty}^{\infty} \frac{\Lambda^{\alpha\beta}(n)}{\omega - k_z v_z - n\Omega_e} \\ & + \sum_{r=+,-} \int d\mathbf{p} \sum_{n=-\infty}^{\infty} \frac{\Lambda_{S,r}^{\alpha\beta}(n)}{\omega - k_z v_z - n\Omega_e + r\Omega_\mu} \\ & - 4\pi\mu_e^2 \frac{k_x^2 c^2}{\omega^2} \int d\mathbf{p} \sum_{n=-\infty}^{\infty} J_n^2 \frac{\partial f_0}{\partial \varepsilon} \delta^{\alpha y} \delta^{\beta y} + \Delta^{\alpha} \delta^{\beta y}, \quad (44) \end{aligned}$$

where

$$\begin{aligned} \Lambda^{\alpha\beta}(n) = & \frac{4\pi q_e^2}{\omega} \frac{\partial f_0}{\partial \varepsilon} \Pi_{Cl}^{\alpha\beta}(n) \\ & + 4\pi\mu_e^2 \frac{k_x^2 c^2}{\omega} J_n^2 \frac{\partial f_0}{\partial \varepsilon} \delta^{\alpha y} \delta^{\beta y} + \frac{4\pi q_e \mu_e c}{\omega} \Pi_S^{\alpha\beta}(n) \frac{\partial S_{0z}}{\partial \varepsilon}, \quad (45) \end{aligned}$$

with

$$\hat{\Pi}_{Cl}(n) = \begin{pmatrix} \frac{\Omega_e^2}{k_x^2} n^2 J_n^2 & v_\perp \frac{\Omega_e}{k_x} n J_n J'_n & v_z \frac{\Omega_e}{k_x} n J_n^2 \\ -v_\perp \frac{\Omega_e}{k_x} n J_n J'_n & v_\perp^2 (J'_n)^2 & -v_\perp v_z J_n J'_n \\ v_z \frac{\Omega_e}{k_x} n J_n^2 & v_\perp v_z J_n J'_n & v_z^2 J_n^2 \end{pmatrix} \quad (46)$$

describing the classical part and presented in many textbooks (see for instance [49], [50]), and the tensor

$$\Pi_S^{\alpha\beta}(n) = \begin{pmatrix} 0 & i\Omega_e n J_n^2 & 0 \\ -i\Omega_e n J_n^2 & 2k_x v_\perp J_n J'_n & -ik_x v_z J_n^2 \\ 0 & ik_x v_z J_n^2 & 0 \end{pmatrix} \quad (47)$$

describes the spin evolution leading to the same resonances as the classic evolution $\omega = k_z v_z + n\Omega_e$. The Bessel functions J_n and their derivatives J'_n are used here. In this section all Bessel functions J_n are functions of $k_x v_\perp / \Omega_e$.

Next, present tensors describing spin evolution manifesting itself at shifted resonances $\omega = k_z v_z + n\Omega_e \pm \Omega_\mu$:

$$\hat{\Lambda}_{S+}(n) = \frac{4\pi}{\omega} \frac{\mu_e^2 c^2}{2\omega} J_n^2 \left(\frac{\partial f_0}{\partial \varepsilon} (k_z v_z + n\Omega_e) + \frac{2S_{0z}}{\hbar} \right) \hat{K}, \quad (48)$$

and

$$\hat{\Lambda}_{S-}(n) = \frac{4\pi}{\omega} \frac{\mu_e^2 c^2}{2\omega} J_n^2 \left(\frac{\partial f_0}{\partial \varepsilon} (k_z v_z + n\Omega_e) - \frac{2S_{0z}}{\hbar} \right) (\hat{K})^*, \quad (49)$$

where

$$\hat{K} = \begin{pmatrix} k_z^2 & -ik_z^2 & -k_x k_z \\ ik_z^2 & k_z^2 & -ik_x k_z \\ -k_x k_z & ik_x k_z & k_x^2 \end{pmatrix}, \quad (50)$$

and symbol $*$ means the complex conjugation.

The last term in the dielectric permeability tensor (44) arises as follows

$$\begin{aligned} \Delta = & \frac{4\pi q_e \mu_e c}{\omega^2} \int \sum_{n=-\infty}^{\infty} \frac{\partial S_{0z}}{\partial \varepsilon} \times \\ & \times \{-i\Omega_e n J_n^2, -k_x v_\perp J_n J'_n, -ik_x v_z J_n^2\} d\mathbf{p}. \quad (51) \end{aligned}$$

The x and z projections of vector Δ are equal to zero. The x projection is equal to zero due to explicit summation on n and the z projection is equal to zero due to integration over angles (some details are discussed in the Appendix C). Therefore, the last term in the dielectric permeability tensor (44) gives contribution in element ε^{yy} only. This contribution has the following form: $\Delta_y \delta^{\alpha y} \delta^{\beta y}$, with $\Delta_y = -k_x \frac{4\pi q_e \mu_e c}{\omega^2} \int \frac{\partial S_{0z}}{\partial \varepsilon} v_\perp J_0 J'_0 d\mathbf{p}$.

B. Dielectric permeability tensor for the spin-polarized Fermi step distribution function

The dielectric permeability tensor is obtained above for the general form of isotropic distribution function. In this subsection, a special form of the dielectric permeability tensor is obtained for the spin-1/2 partially polarized 3D electron gas. To this end the equilibrium distribution functions are chosen as follows $f_0(p) = [\vartheta(p_{F\uparrow} - p) + \vartheta(p_{F\downarrow} - p)]/(2\pi\hbar)^3$ and $S_{0z}(p) = [\vartheta(p_{F\uparrow} - p) - \vartheta(p_{F\downarrow} - p)]/(2\pi\hbar)^3$, where $p_{Fs} = (6\pi^2 n_{0s})^{1/3} \hbar$.

As it follows from equation (2), the Fermi spin current is related to difference between concentrations of electrons with different spin projections. In kinetic description such difference comes from S_{0z} which is proportional to difference of the Fermi steps of the spin-up and spin-down electrons.

The derivatives of the distribution functions are proportional to the Dirac delta function $\delta(p - p_{Fs})$, since $\partial\vartheta(p_{Fs} - p)/\partial p = -\delta(p - p_{Fs})$. Therefore, the integrals over the module of the momentum module can be easily calculated. As the result we find:

$$\varepsilon^{\alpha\beta} = \delta^{\alpha\beta} - \sum_{s=\uparrow,\downarrow} \int \sin\theta d\theta \sum_{n=-\infty}^{\infty} \left[\frac{\tilde{\Lambda}^{\alpha\beta}(n, s)}{\omega - k_z v_{Fs} \cos\theta - n\Omega_e} + \frac{mp_{Fs} \mu_e^2 c^2}{\pi \hbar^3 2\omega^2} \sum_{r=+,-} \frac{J_n^2(k_z v_{Fs} \cos\theta + n\Omega_e) \kappa_r^{\alpha\beta}}{\omega - k_z v_{Fs} \cos\theta - n\Omega_e + r\Omega_\mu} \right. \\ \left. - \frac{1}{\pi \hbar^3} \frac{\mu_e^2 c^2}{\hbar \omega^2} \sum_{r=+,-} \int_0^{p_{Fs}} p^2 dp \frac{r(-1)^{i_s} J_n^2 \kappa_r^{\alpha\beta}}{\omega - k_z v_z - n\Omega_e + r\Omega_\mu} - \frac{mp_{Fs}}{\pi \hbar^3} \left(\mu_e^2 \frac{k_x^2 c^2}{\omega^2} J_n^2 + \frac{q_e \mu_e k_x c}{\omega^2} v_{Fs} \sin\theta J_n J'_n \right) \delta^{\alpha y} \delta^{\beta y} \right], \quad (52)$$

where $\kappa_+^{\alpha\beta} = K^{\alpha\beta}$, $\kappa_-^{\alpha\beta} = (K^{\alpha\beta})^*$, $i_\uparrow = 0$, $i_\downarrow = 1$, and

$$\tilde{\Lambda}^{\alpha\beta}(n, s) = \frac{3\omega_{Ls}^2}{2\omega v_{Fs}^2} \Pi_{Cl}^{\alpha\beta}(n, s) + m^2 v_{Fs} \left(\frac{\mu_e^2 k_x^2 c^2}{\pi \hbar^3 \omega} J_n^2 \delta^{\alpha y} \delta^{\beta y} + \frac{q_e \mu_e}{\pi \hbar^3} (-1)^{i_s} \frac{c}{\omega} \Pi_S^{\alpha\beta}(n, s) \right), \quad (53)$$

with

$$\hat{\Pi}_{Cl}(n, s) = \begin{pmatrix} \frac{\Omega_e^2}{k_x^2} n^2 J_n^2 & v_{Fs} \sin\theta \frac{\Omega_e}{k_x} n J_n J'_n & v_{Fs} \cos\theta \frac{\Omega_e}{k_x} n J_n^2 \\ -v_{Fs} \sin\theta \frac{\Omega_e}{k_x} n J_n J'_n & v_{Fs}^2 \sin^2\theta (J'_n)^2 & -v_{Fs}^2 \sin\theta \cos\theta J_n J'_n \\ v_{Fs} \cos\theta \frac{\Omega_e}{k_x} n J_n^2 & v_{Fs}^2 \sin\theta \cos\theta J_n J'_n & v_{Fs}^2 \cos^2\theta J_n^2 \end{pmatrix} \quad (54)$$

which has structure similar to well-known from textbooks [50], but it separately describes electrons with spin-up and spin-down, and the tensor

derivation of the dielectric permeability tensor have been presented.

$$\Pi_S^{\alpha\beta}(n, s) =$$

Acknowledgments

$$\begin{pmatrix} 0 & i\Omega_e n J_n^2 & 0 \\ -i\Omega_e n J_n^2 & 2k_x v_{Fs} \sin\theta J_n J'_n & -ik_x v_{Fs} \cos\theta J_n^2 \\ 0 & ik_x v_{Fs} \cos\theta J_n^2 & 0 \end{pmatrix} \quad (55)$$

describes the spin evolution leading to the same resonances as the classic evolution $\omega = k_z v_{Fs} \cos\theta + n\Omega_e$. Here all Bessel functions J_n are functions of $k_x v_{Fs} \sin\theta / \Omega_e$.

V. CONCLUSION

A single fluid spin-1/2 quantum kinetics of electrons has been applied for a derivation of the dielectric permeability tensor of spin polarized electron gas. This model consists of two kinetic equations for each species of particles: a generalization of the Vlasov equation containing the spin-spin interaction as an additional term with the spin distribution function and a vector equation for the spin distribution function. Necessity of the application of this model with the spin polarized equilibrium distribution functions follows from the existence of the thermal part of spin current or the Fermi spin current for degenerate electrons which affects properties of transverse waves including the spin-plasma waves. More detailed description of spin current effects requires a kinetic model which has been presented here. Necessary details of the solution of linearized set of kinetic equation required for the

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VI. APPENDIX A: LINEAR PART OF THE DISTRIBUTION FUNCTIONS FOR GENERALIZED EQUILIBRIUM SPIN DISTRIBUTION FUNCTIONS

In equilibrium the set of kinetic equations (9) and (10) can be presented in the following form

$$\partial_\varphi f_{0e\uparrow} = 0, \quad \partial_\varphi f_{0e\downarrow} = 0, \quad \partial_\varphi f_{0i} = 0, \quad (56)$$

and

$$\partial_\varphi S_{0e,x} = S_{0e,y}, \quad \partial_\varphi S_{0e,y} = -S_{0e,x}, \quad (57)$$

where time and space derivatives of the distribution functions are equal to zero, the equilibrium electric field is equal to zero. The equilibrium magnetic field is equal to the external field: $\mathbf{B}_0 = \mathbf{B}_{ext} = B_0 \mathbf{e}_z$.

Equations (57) give the general form of dependence of equilibrium spin distribution functions on momentum $S_{0x} = C(p_\parallel, p_\perp) \cos(\varphi + \varphi_0)$, $S_{0y} = C(p_\parallel, p_\perp) \sin(\varphi + \varphi_0)$. Constant C can be equal to zero. It gives the isotropic equilibrium considered in the paper. For isotropic f_0 and

S_{0z} and nonzero constant C , functions S_{0x} and S_{0y} can be presented in the following form [28]:

$$S_{0x} = \frac{1}{(2\pi\hbar)^3} \left(\Theta(p_{F\uparrow} - p) - \Theta(p_{F\downarrow} - p) \right) \cos(\varphi + \varphi_0), \quad (58)$$

$$S_{0y} = \frac{1}{(2\pi\hbar)^3} \left(\Theta(p_{F\uparrow} - p) - \Theta(p_{F\downarrow} - p) \right) \sin(\varphi + \varphi_0). \quad (59)$$

In equation (17), there is a change of the last term in the following way: $\delta B_z(\mathbf{k}\nabla_{\mathbf{p}})S_{0z} \rightarrow \delta B_\beta(\mathbf{k}\nabla_{\mathbf{p}})S_{0\beta} = \delta B_\beta[(\mathbf{k}\mathbf{p})\partial_p S_{0\beta}/p + \varepsilon^{\beta\gamma z} S_{0\gamma} \varepsilon^{\mu\nu z} k_\mu p_\nu]$. So, solution (22) is modified to

$$\delta f = \frac{1}{\Omega_e} \int_{C_0}^\varphi \left(q_e(\mathbf{v} \cdot \delta \mathbf{E}) \frac{\partial f_0}{\partial \varepsilon} + \imath \mu_e(\mathbf{k} \cdot \mathbf{v}) \left(\delta \mathbf{B} \cdot \frac{\partial \mathbf{S}_0}{\partial \varepsilon} \right) + \imath \mu_e([\delta \mathbf{B}, \mathbf{S}_0]_z) \cdot ([\mathbf{k}, \mathbf{p}]_z) \right) \exp \left(\imath \int_\varphi^{\varphi'} \frac{(\omega - \mathbf{k} \cdot \mathbf{v} |_{\varphi''})}{\Omega_e} d\varphi'' \right) d\varphi'. \quad (60)$$

Equation (18) contains full vector \mathbf{S}_0 . So, changing the explicit form of \mathbf{S}_0 we have the spin distribution function evolution equation in the generalized regime. Equations (61)-(63) are modified to

$$\begin{aligned} & \Omega_e \partial_\varphi \delta S_x + \imath(\omega - \mathbf{k} \cdot \mathbf{v}) \delta S_x + \Omega_\mu \delta S_y \\ &= q_e \delta \mathbf{E} \cdot \nabla_{\mathbf{p}} S_{0x} + \imath \mu_e(\mathbf{k} \cdot \nabla_{\mathbf{p}}) f_0 \delta B_x + \frac{2\mu_e}{\hbar} S_{0z} \delta B_y - \frac{2\mu_e}{\hbar} S_{0y} \delta B_z - S_{0y}(mv^2 \delta B_z - mv_z(\mathbf{v} \delta \mathbf{B})), \end{aligned} \quad (61)$$

$$\begin{aligned} & -\Omega_\mu \delta S_x + \Omega_e \partial_\varphi \delta S_y + \imath(\omega - \mathbf{k} \cdot \mathbf{v}) \delta S_y \\ &= q_e \delta \mathbf{E} \cdot \nabla_{\mathbf{p}} S_{0y} + \imath \mu_e(\mathbf{k} \cdot \nabla_{\mathbf{p}}) f_0 \delta B_y - \frac{2\mu_e}{\hbar} S_{0z} \delta B_x + \frac{2\mu_e}{\hbar} S_{0x} \delta B_z + S_{0x}(mv^2 \delta B_z - mv_z(\mathbf{v} \delta \mathbf{B})), \end{aligned} \quad (62)$$

and

$$\Omega_e \partial_\varphi \delta S_z + \imath(\omega - \mathbf{k} \cdot \mathbf{v}) \delta S_z = q_e(\delta \mathbf{E} \cdot \nabla_{\mathbf{p}}) S_{0z} + \imath \mu_e(\mathbf{k} \cdot \nabla_{\mathbf{p}}) f_0 \delta B_z + \frac{2\mu_e}{\hbar} (\delta B_x S_{0y} - \delta B_y S_{0x}) \quad (63)$$

(17) and (18) to equations (61)-(63).

Therefore, solution (64) is modified to

$$\delta S_z = \frac{1}{\Omega_e} \int_{C_3}^\varphi \left(q_e(\mathbf{v} \cdot \delta \mathbf{E}) \frac{\partial S_{0z}}{\partial \varepsilon} + \imath \mu_e(\mathbf{k} \cdot \mathbf{v}) \delta B_z \frac{\partial f_0}{\partial \varepsilon} + \frac{2\mu_e}{\hbar} (\delta B_x S_{0y} - \delta B_y S_{0x}) \right) \exp \left(\imath \int_\varphi^{\varphi'} \frac{(\omega - \mathbf{k} \cdot \mathbf{v} |_{\varphi''})}{\Omega_e} d\varphi'' \right) d\varphi', \quad (64)$$

In both regimes solutions for δS_x and δS_y are presented by equations (61) and (62), but in the generalized case functions Π_x and Π_y have the following form found from equations (61) and (62)

$$\Pi_x = \frac{1}{\Omega_e} \exp \left(\imath \int_C^\varphi \frac{1}{\Omega_e} (\omega - \mathbf{k} \cdot \mathbf{v} |_{\varphi'}) d\varphi' \right) \left(\imath \mu_e(\mathbf{k} \cdot \nabla_{\mathbf{p}}) f_0 \delta B_x + \frac{2\mu_e}{\hbar} S_{0z} \delta B_y - \frac{2\mu_e}{\hbar} S_{0y} \delta B_z - S_{0y}(mv^2 \delta B_z - mv_z(\mathbf{v} \delta \mathbf{B})) \right), \quad (65)$$

and

$$\Pi_y = \frac{1}{\Omega_e} \exp \left(\imath \int_C^\varphi \frac{1}{\Omega_e} (\omega - \mathbf{k} \cdot \mathbf{v} |_{\varphi'}) d\varphi' \right) \left(\imath \mu_e(\mathbf{k} \cdot \nabla_{\mathbf{p}}) f_0 \delta B_y - \frac{2\mu_e}{\hbar} S_{0z} \delta B_x + \frac{2\mu_e}{\hbar} S_{0x} \delta B_z + S_{0x}(mv^2 \delta B_z - mv_z(\mathbf{v} \delta \mathbf{B})) \right). \quad (66)$$

VII. APPENDIX B: AN INTERMEDIATE FORM OF THE CONDUCTIVITY TENSOR

More explicit form of the conductivity tensor caused by the current is obtained at substituting of the distribution

function δf

$$\sigma_1^{\alpha\beta}(\omega) = \frac{q_e}{m} \frac{1}{\Omega_e} \int d\mathbf{p} p^\alpha \int_{C_0}^\varphi d\varphi' \left(q_e v^\beta |_{\varphi'} \frac{\partial f_0}{\partial \varepsilon} \right)$$

$$+i\mu_e(\mathbf{k} \cdot \mathbf{v})|_{\varphi'} \frac{k_x c}{\omega} \delta^{y\beta} \frac{\partial S_{0z}}{\partial \varepsilon} \exp\left(i \int_{\varphi}^{\varphi'} \frac{\omega - \mathbf{k} \cdot \mathbf{v}|_{\varphi''}}{\Omega_e} d\varphi''\right). \quad \text{and} \quad (67)$$

The conductivity tensor caused by the curl of magnetization has a complicate structure. Hence, it is splitted on three parts:

$$\sigma_2^{x\beta}(\omega) \delta E_\beta = -i\mu_e c \int k_z \delta S_y d\mathbf{p}, \quad (68)$$

$$\sigma_2^{y\beta}(\omega) \delta E_\beta = i\mu_e c \int (k_z \delta S_x - k_x \delta S_z) d\mathbf{p}, \quad (69)$$

$$\sigma_2^{z\beta}(\omega) \delta E_\beta = i\mu_e c \int k_x \delta S_y d\mathbf{p}. \quad (70)$$

Explicit forms of functions (68)-(70) appear at substitution of the distribution functions δS_x , δS_y , and δS_z . Thus, the following expression can be found for $\sigma_2^{x\beta}(\omega)$:

$$\begin{aligned} \sigma_2^{x\beta}(\omega) \delta E_\beta = & -\mu_e^2 \frac{k_z c^2}{2\Omega_e \omega} \int \left\{ \int_{C_1}^{\varphi} \exp\left(\frac{i\Omega_\mu}{\Omega_e}(\varphi - \varphi')\right) \exp\left(i \int_{\varphi}^{\varphi'} \frac{\omega - \mathbf{k} \cdot \mathbf{v}|_{\varphi''}}{\Omega_e} d\varphi''\right) \left[\left(-(\mathbf{k} \cdot \mathbf{v})|_{\varphi'} \frac{\partial f_0}{\partial \varepsilon} + i \frac{2S_{0z}}{\hbar} \right) k_z \delta E_y \right. \right. \\ & + \left. \left((\mathbf{k} \cdot \mathbf{v})|_{\varphi'} \frac{\partial f_0}{\partial \varepsilon} + \frac{2S_{0z}}{\hbar} \right) (k_z \delta E_x - k_x \delta E_z) \right] d\varphi' + \int_{C_2}^{\varphi} \exp\left(\frac{i\Omega_\mu}{\Omega_e}(\varphi' - \varphi)\right) \exp\left(i \int_{\varphi}^{\varphi'} \frac{\omega - \mathbf{k} \cdot \mathbf{v}|_{\varphi''}}{\Omega_e} d\varphi''\right) \times \\ & \times \left[\left((\mathbf{k} \cdot \mathbf{v})|_{\varphi'} \frac{\partial f_0}{\partial \varepsilon} + i \frac{2S_{0z}}{\hbar} \right) k_z \delta E_y + \left((\mathbf{k} \cdot \mathbf{v})|_{\varphi'} \frac{\partial f_0}{\partial \varepsilon} - \frac{2S_{0z}}{\hbar} \right) (k_z \delta E_x - k_x \delta E_z) \right] d\varphi' \Big\} d\mathbf{p}. \end{aligned} \quad (71)$$

Neglecting the anomalous magnetic moment of electron we have $\Omega_\mu = \Omega_e$. Hence, $\sigma_2^{x\beta}(\omega)$ consists of two parts proportional to $e^{i(\varphi - \varphi')}$ or $e^{-i(\varphi - \varphi')}$. Elements $\sigma_2^{y\beta}(\omega)$ have the following explicit form:

$$\begin{aligned} \sigma_2^{y\beta}(\omega) \delta E_\beta = & i\mu_e^2 \frac{k_z c^2}{2\Omega_e \omega} \int \left\{ \int_{C_1}^{\varphi} \exp\left(\frac{i\Omega_\mu}{\Omega_e}(\varphi - \varphi')\right) \exp\left(i \int_{\varphi}^{\varphi'} \frac{\omega - \mathbf{k} \cdot \mathbf{v}|_{\varphi''}}{\Omega_e} d\varphi''\right) \left[\left(-(\mathbf{k} \cdot \mathbf{v})|_{\varphi'} \frac{\partial f_0}{\partial \varepsilon} + i \frac{2S_{0z}}{\hbar} \right) k_z \delta E_y \right. \right. \\ & + \left. \left((\mathbf{k} \cdot \mathbf{v})|_{\varphi'} \frac{\partial f_0}{\partial \varepsilon} + \frac{2S_{0z}}{\hbar} \right) (k_z \delta E_x - k_x \delta E_z) \right] d\varphi' + \int_{C_2}^{\varphi} \exp\left(\frac{i\Omega_\mu}{\Omega_e}(\varphi' - \varphi)\right) \exp\left(i \int_{\varphi}^{\varphi'} \frac{\omega - \mathbf{k} \cdot \mathbf{v}|_{\varphi''}}{\Omega_e} d\varphi''\right) \times \\ & \times \left[\left(-(\mathbf{k} \cdot \mathbf{v})|_{\varphi'} \frac{\partial f_0}{\partial \varepsilon} - i \frac{2S_{0z}}{\hbar} \right) k_z \delta E_y + \left(-(\mathbf{k} \cdot \mathbf{v})|_{\varphi'} \frac{\partial f_0}{\partial \varepsilon} + \frac{2S_{0z}}{\hbar} \right) (k_z \delta E_x - k_x \delta E_z) \right] d\varphi' \\ & - k_x \int_{C_3}^{\varphi} \left(\frac{q_e}{\mu_e} (\mathbf{v} \cdot \delta \mathbf{E})|_{\varphi'} \frac{\partial S_{0z}}{\partial \varepsilon} + (\mathbf{k} \cdot \mathbf{v})|_{\varphi'} \frac{k_x c}{\omega} \delta E_y \frac{\partial f_0}{\partial \varepsilon} \right) \exp\left(i \int_{\varphi}^{\varphi'} \frac{\omega - \mathbf{k} \cdot \mathbf{v}|_{\varphi''}}{\Omega_e} d\varphi''\right) d\varphi' \Big\} d\mathbf{p}. \end{aligned} \quad (72)$$

Similar to $\sigma_2^{x\beta}(\omega)$, $\sigma_2^{y\beta}(\omega)$ consists of two parts proportional to $e^{i(\varphi - \varphi')}$ or $e^{-i(\varphi - \varphi')}$. Elements $\sigma_2^{z\beta}$ can be simply presented via $\sigma_2^{x\beta}$

$$\sigma_2^{z\beta}(\omega) = -\frac{k_x}{k_z} \sigma_2^{x\beta}(\omega) \quad (73)$$

in accordance with equations (68) and (70).

VIII. APPENDIX C: CALCULATION OF SEVERAL ELEMENTS IN DIELECTRIC PERMEABILITY TENSOR

For calculation of z-projection of Δ (51) we need to consider integral over θ . Dependence on θ comes from $v_z J_n^2(v_\perp)$. So, we have integral $I_n = \int \sin \theta d\theta \cos \theta J_n^2(a \sin \theta)$, where $a \equiv k_x v_{Fs} / \Omega_e$. Using

$J_{-n}(z) = (-1)^n J_n(z)$ and $J_{-n}^2(z) = J_n^2(z)$ we reduce our calculations to nonnegative Bessel functions $n \geq 0$. Explicit integration can be performed at the application of series expansion of the Bessel function:

$$J_n(z) = \sum_{k=0}^{\infty} \frac{(-1)^k z^n (z/2)^{2k}}{2^n k! \Gamma(n+k+1)}. \quad (74)$$

Integrating we obtain that $I_n \sim \sin^{2k+2l+2n+2} \theta |_{\theta=0} = 0$, where $k, l, n \geq 0$ and l is used in expansion of the second Bessel function presented under the integral. x -projection of Δ is proportional to nJ_n^2 we can explicitly calculate sum of these terms and find that it is equal to

zero:

$$\begin{aligned} \sum_{n=-\infty}^{\infty} nJ_n^2 &= \sum_{n=1}^{\infty} nJ_n^2 + \sum_{n=-1}^{-\infty} nJ_n^2 \\ &= \sum_{n=1}^{\infty} (nJ_n^2 + (-n)J_{-n}^2) = 0, \end{aligned} \quad (75)$$

where we have used $J_{-n}(z) = (-1)^n J_n(z)$. Similarly, applying equation $J_n'(z) = J_{n-1}(z) - \frac{n}{z} J_n(z)$, we find $\sum_{n=-\infty}^{\infty} J_n J_n' = J_0 J_0' = 0$. It simplifies y -projection of Δ .

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